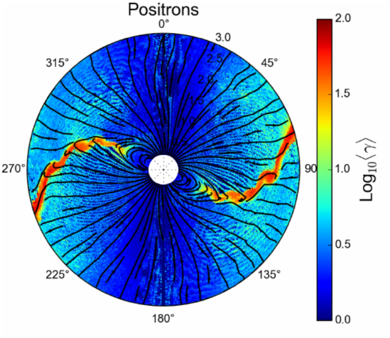
A look at the numerical simulations codes for isolated pulsars

Fabrice Mottez

LUTH - Obs. Paris-Meudon - CNRS - Univ. Paris Diderot

MODE 2019 Orleans

The question is simple and basic

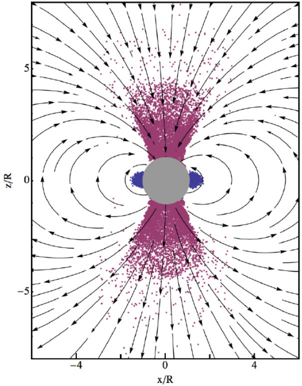


Lorentz factor of positrons in a Msphere, PSR obliquity 30^o [Cerutti + 2016]

A system with clearly defined ingredients

- A conducting sphere $T \sim 10^6$ K;
- Particles can be extracted from it;
- Fast rotation;
- Strong magnetic field near the sphere ;
- Compute the electromagnetic field (Maxwell's equations);
- Move the (relativistic) particles (Lorentz + radiation reaction forces);
- Compute the radiation (Wiennert eqs)
- Compute e^- e^+ pair creations (QED's equations);
- Near the star, take GR effects into account;

Tools for answering that question

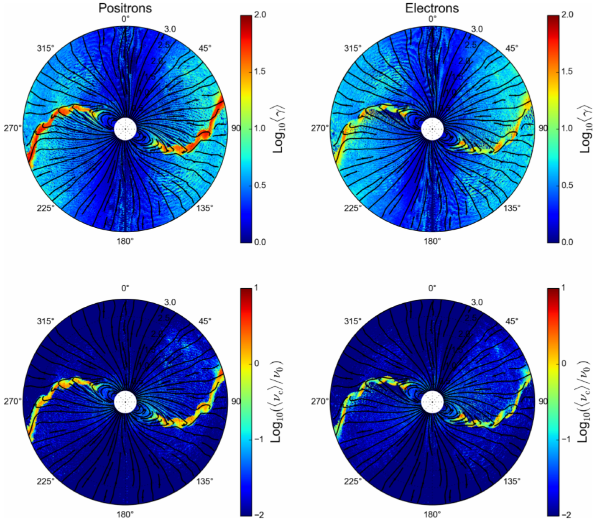


electrosphere = when no pairs are created
[Spitkovski + 2014]

Particle In Cell codes

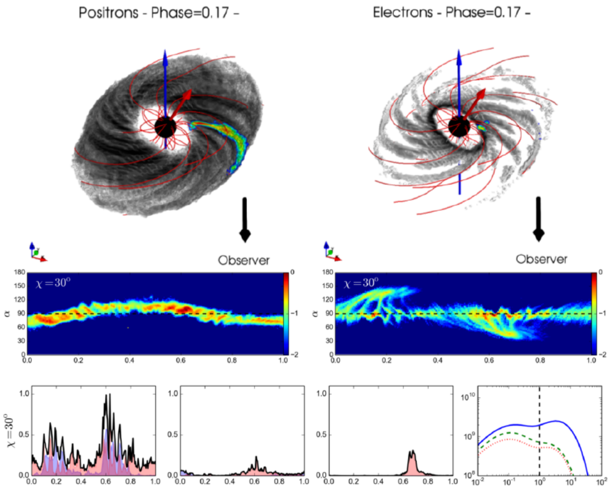
- Compute the electromagnetic field on a grid from ρ_i and j_i defined on that grid;
- In that field, move a large number of macro-particles;
- From the new position of the particles, deposit ρ_i and j_i onto the grid;
- From the trajectories of the particles, compute their radiation (synchrotron, curvature radiation);
- Move the photons and use them to compute pair creations ;
- Set appropriate inner (star) and outer (far end) boundary conditions
- Set appropriate initial conditions

A success: Magnetosphere model



Lorentz factor and synchrotron radiation critical frequency $\sim B_\perp \gamma^2$ [Cerutti + 2016]

A success: High energy radiation map



Radiation by positron and electrons, corresponding radiation maps, and pulse profiles at 3 different viewing angles + spectrum [Cerutti + 2016]

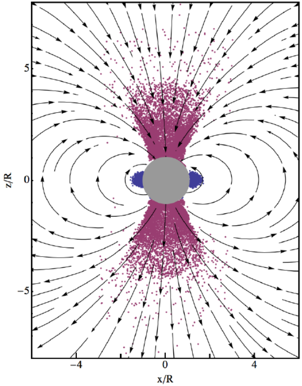
A list of PIC simulations of isolated pulsars

Table 1 Complete list of pulsar magnetosphere modelling using global PIC simulations in order of publication date. "GR" stands for General Relativistic, "Rad." stands for Radiation, and "Part." for Particle.

References	Inclination	Part. injection	Extra physics
Philippov & Spitkovsky (2014) [6]	Aligned	Volume	
Chen & Beloborodov (2014) [7]	Aligned	Pair creation	
Cerutti et al. (2015) [8]	Aligned	Surface	
Belyaev (2015) [9]	Aligned	$\mathbf{E} \cdot \mathbf{B} \neq 0$	
Philippov et al. (2015) [10]	Oblique	Pair creation	
Philippov et al. (2015) [11]	Aligned	Pair creation	GR corrections
Cerutti et al. (2016) [12] [13]	Oblique	Surface	Rad. & polarization
Cerutti & Philippov (2017) [14]	Oblique	Surface	
Philippov & Spitkovsky (2018) [15]	Oblique	Pair creation	GR & radiation
Kalapotharakos et al. (2018) [16]	Oblique	Volume	Radiation
Brambilla et al. (2018) [17]	Oblique	Volume	

[Benoît Cerutti 2018]

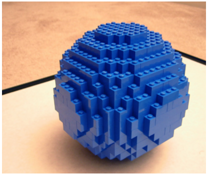
How to partition space: grid shape



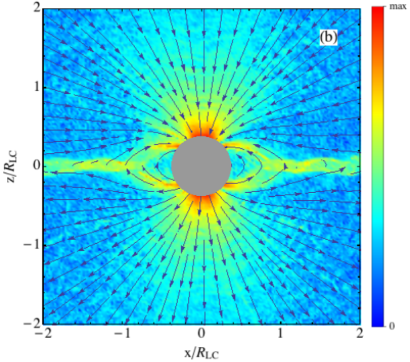
3D code TRISTAN. Cartesian grid, 1000^3 cells, $R_* = 50\Delta x$. No pair creation. [Spitkovski 2014+]

Cartesian + finite differences

- 🙂 3D Cartesian grid like with 3D code TRISTAN [Spitkovsky since 2005].
- Uniform resolution Δx.
- Solve field equations with finite differences.
- 🙁 The internal sphere (=neutron star surface or injection surface) may look like Lego TM (not a big issue, theses bricks are fun).



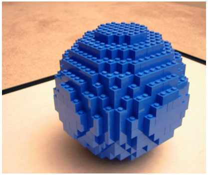
How to partition space: grid shape



3D code TRISTAN. Cartesian grid, 1000^3 cells, $R_* = 50\Delta x$. Massive pair injection. [Philippov & Spitkovski 2014]

Cartesian + finite differences

- 3D Cartesian grid like with 3D code TRISTAN [Spitkovsky since 2005].
- Uniform resolution Δx.
- Solve field equations with finite differences.
- the internal sphere (=neutron star surface or injection surface) looks like Lego (TM) (not a big issue, theses bricks are fun).



How to partition space: grid shape. How to preserve scales

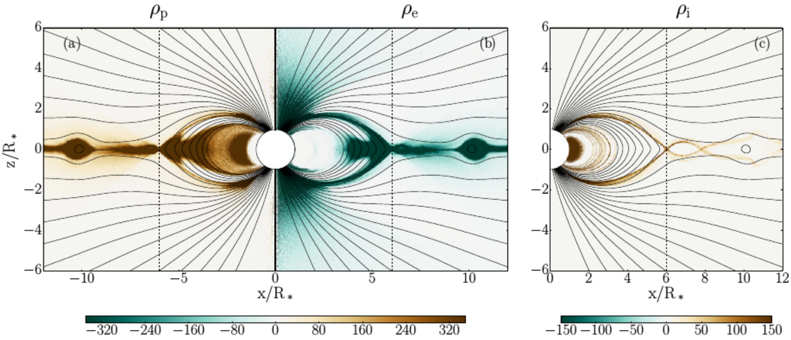
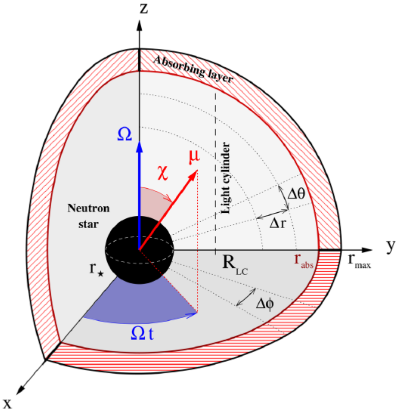


Figure 2. Charge densities of (a) positrons, (b) electrons, and (c) ions.

- 2D Cartesian grid [Chen 2014], Uniform resolution Δx
- ☹ Must resolve $\lambda = c/\omega_{pe}$, and $R_*/\lambda \sim 100$.
- Then $R_{LC}/R_* \sim 5 - 10$, i.e $P = 1$ ms. $R_{out} = 30R_* >> R_{LC}$
- ☹ Everything is smaller than in real world, but hierarchy of scales is preserved.
- 😕 We can hope that the simulations are qualitatively good.

How to partition space: grid shape

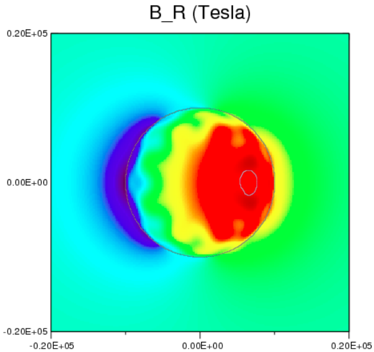


Simulation domain and Yee-lattice in ZELTRON code [Cerutti + 2016]

Spherical lattice + finite differences

- ☺ Spherical Yee grid like in code ZELTRON [Cerutti + 2016].
- Kind of finite differences.
- Use Cartesian grid for deposition of ρ and $\vec{j}$ from particles.
- Remapping between the two grids (interpollation)
- ☺ Uniform resolution in θ and ϕ, logaritmically spaced in r (better than uniform).

How to partition space: grid shape

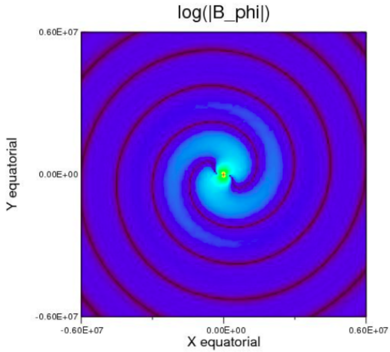


A rotating multipole in vacuum [Mottez]. Multi-domains spectral method. Near the star (maked with a gray circle).

Imbricated spherical lattices + spectral methods

- Decompose the fields in sherical harmonics in θ and ϕ
- Decompose the fields in Chebychev polynomials in r on each domain.
- Connect the domains. They can have different (arbitrary) sizes in r.
- Jump many times between real and spectral représentation.
- Use Cartesian coordinates to push particles, but no Cartesian grid needed.
- Requires small number of nodes (32/domain) if no sharp gradients. (OK for PSR wind, not for shockwaves.)
- No published PIC simulation with this method.

How to partition space: grid shape



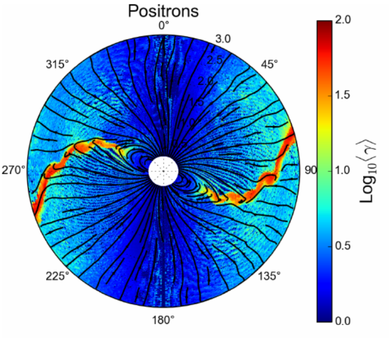
A rotating multipole in vacuum [Mottez].
Multi-domains spectral method. Up to 300 R_*.
Also mastered by J. Petri.

Imbricated spherical lattices + spectral methods

- Decompose the fields in sherical harmonics in θ and ϕ
- Decomposethe fields in Chebychev polynomials in r on each domain.
- Connect the domains. They can have different (arbitrary) sizes in r.
- Jump many times between real and spectral représentation.
- Use Cartesian coordinates to push particles, but no Cartesian grid needed.
- Requires small number of nodes (32/domain) if no sharp gradients. (OK for PSR wind, not for shockwaves.)
- No published PIC simulation with this method.

1

Particle motion : the expensive part of PIC simulations



Lorentz factor of positrons in a Msphere, PSR obliquity 30^o [Cerutti + 2016]

Full trajectories

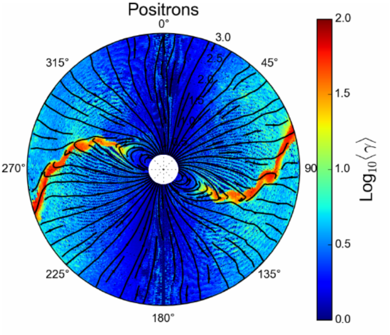
$$m\frac{d\gamma\vec{v}}{dt} = \frac{q}{m}(\vec{E} + v \times \vec{B}) + \vec{g}$$

and $\vec{g}$ is the radiation reaction force

$$\vec{g} = \frac{2}{3}r_e^2[(\vec{E} + \vec{\beta} \times \vec{B}) \times \vec{B} + (\vec{\beta}.\vec{E})\vec{E}]$$

$$-\frac{2}{3}r_e^2\gamma^2[(\vec{E} + \vec{\beta} \times \vec{B})^2 - (\vec{\beta}.\vec{E})^2]\vec{\beta}$$

Particle motion : the expensive part of PIC simulations



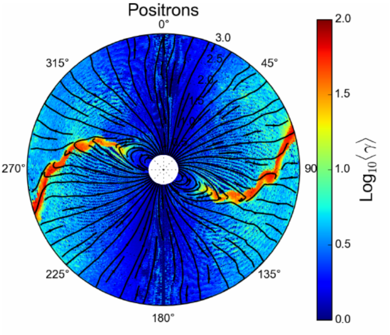
Lorentz factor of positrons in a Msphere, PSR obliquity 30^o. The particle full dynamics is taken into account. [Cerutti + 2016]

Full trajectories

$$m\frac{d\gamma\vec{v}}{dt} = \frac{q}{m}(\vec{E} + v \times \vec{B}) + \vec{g}$$

- ☺ Implemented in code ZELTRON (figure).
- ☺ A largely used implicit alogrithm published by Jean-Luc Vay [Phys. of Plasmas 2008]
- ☺ Contains all the (classical) physics.
- ☹ Time resolution $qB\Delta t/m << 1$ is very constraining. Δt is the same everywhere and is small.
- 😐 Scaling : reduce magnetic field B (by orders of magnitude).
- 😐 Particle energies are consequently reduced.

Particle motion : implicit solvers and approximations based on physics



Lorentz factor of positrons in a Msphere, PSR obliquity 30^o [Cerutti + 2016]

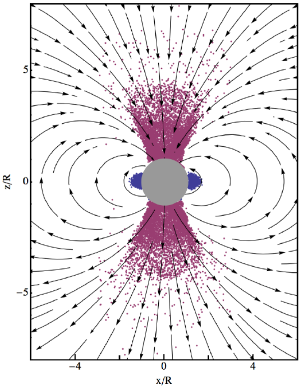
Simplified trajectories

- J. L Vay [2008] is a good start for modified alogorithms.
- Solve equation of motion with implicit algorithm also for positions. Petri [J. Plas. Phys. 2018]
- A potentially inexpensive approximation : $v \sim c$.
- Guiding centre approximation in the strong magnetic fields regions.

Injection of plasma into the simulation

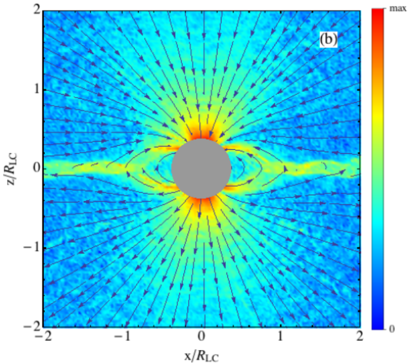
Primary particles

- Primary particles are those which come from the star, considered as a co-rotating plasma.
- No energy required to extract electrons. Less clear with ions.
- Various vays of injecting the primary electrons and ions.
- The primary particle radiate gamma rays, that can trigger a pair creation cascade [Sturrock 1970].



An electrosphere is made only of primary particles [Philippov & Spitkovski 2014]

Injection of plasma into the simulation



No primaries, pairs injected at fixed rate where B above a critical arbitrary value [Philippov & Spitkovski 2014]

Electron-positron pair plasma

- Ideally: compute gamma rays form accelerated particles and gamma+gamma or Gamma+B intercation for pair production.Very expensive, not done yet.

Injection of plasma into the simulation

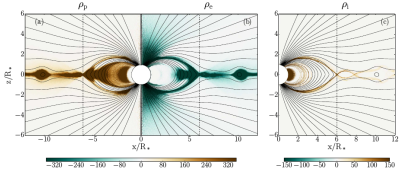


Figure 2. Charge densities of (a) positrons, (b) electrons, and (c) ions.

No primaries, pairs injected where particle above an energy threshold [Chen & Beloborov 2014]

Electron-positron pair plasma

- Inject no primaries. Pairs are injected at a fixed rate if $B > B_{critical}$. Several ad-hoc parameters [Philippov & Spitkovski 2014].
- Associate a pair creation when an electron reaches a threshold energy.
- ☹ Critical energy is scaled down from 10^6 (real pulsars) to 20 (simulations).[Chen & Beloborov 2014]
- Uniform injection of pairs in a thin layer close to the star, at a fixed rate, with poloidal velocity 0.5 c and co-rotating. [Cerutti + 2015]

A conclusion with a simplistic message

PIC simulations are great

- Global view of magnetosphere + wind of millisecond pulsars.
- Describe the acceleration regions and the effect of electric currents on magnetic fields.
- Time dependent models, can describe dynamic processes such as magnetic reconnexion.
- Can provide radiation maps.

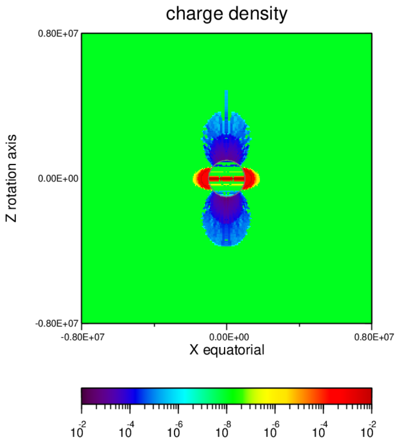
Their limits in 2019

- Only very fast millisecond pulsars. Light cylinder distance $< 5R_*$, $P_* \sim 1$ ms.
- Reduced magnetic fields.
- Reduced acceleration $\gamma \sim 10^2$ instead of 10^6.
- Pair creation is ad-hoc. The PIC simulation cannot yet evaluate the efficiency of the process.
- Not yet ready for quantitative analysis of data.

Need to search for simulations methods allowing for real values of P_*, B_*, γ, pair creation etc.

La critique est aisée, mais l'art est difficile

[Philippe Néricault, dit Destouches, 1732]



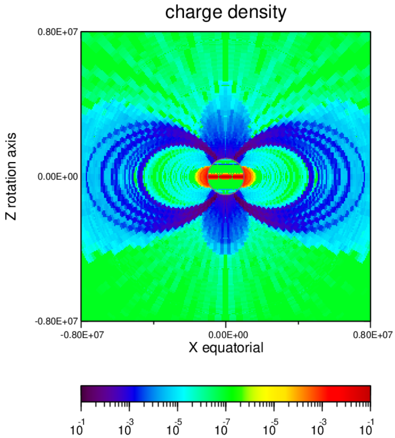
A first attempt of static electrosphere, $P_* = 5$ ms, $B_* = 10^9$ G (iteration 13) [Mottez, in *panic mode* for MODE]

My work for a new algorithm, some progress...

- Not really PIC, more a kind of Vlasov algorithm.
- For faster simulation, I look for a *static* model.
- Primaries injected from the NS star surface with a low energy.
- Grid: spherical, not finite differences but spectral method for Maxwell eqs.
- Motion: relativistic particle guiding centre where B is high. Not yet radiation reaction force. No GR.
- No pair creation yet, no computation of radiation yet.
- Very preliminary, no publication. More next time .

La critique est aisée, mais l'art est difficile

[Philippe Néricault, dit Destouches, 1732]



Oups, my model has not converged yet (iteration 21) [Mottez, in *panic mode* for MODE]

...but still preliminary

- Not really PIC, more a kind of Vlasov algorithm.
- ☹ For faster simulation, I look for a *static* model.
- Primaries injected from the NS star surface with a low energy.
- Grid: spherical, not finite differences but spectral method for Maxwell eqs.
- Motion: relativistic particle guiding centre where B is high. Not yet radiation reaction force. No GR.
- No pair creation yet, no computation of radiation yet.
- Very preliminary, no publication. More next time ☺.