

New constraints on the equation of state of dense nuclear matter using thermal emission of neutron stars

Nicolas Baillot d'Etivaux - IPN Lyon

Workshop MODE2019

Orléan - april, 8th, 2018

in collaboration with :

Jérôme Margueron - IPN Lyon

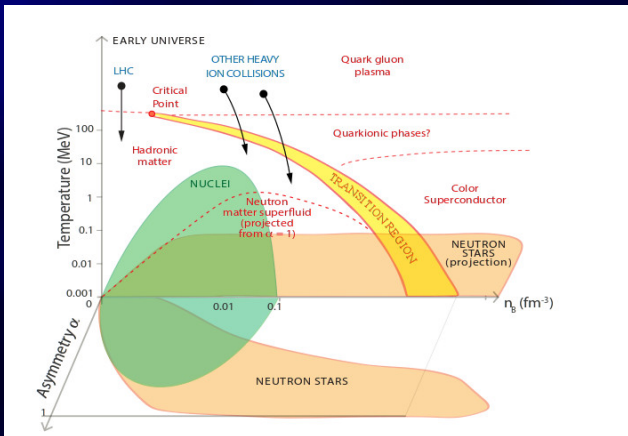
Sébastien Guillot - IRAP Toulouse

Natalie Webb - IRAP Toulouse

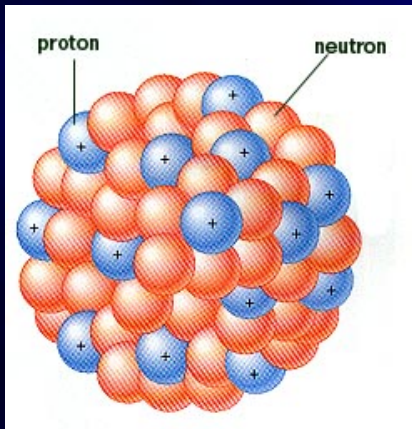
Hubert Hansen - IPN Lyon

Neutron stars are best laboratory for very high densities :

- Hadronic matter at very high density / isospin asymmetry
- **Phase transition ?**



"One Equation of State (EoS) to rule them all..."

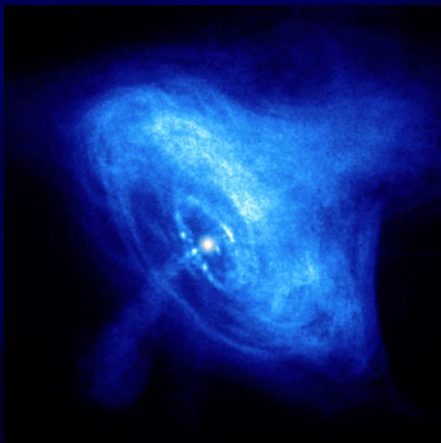


$\simeq 10^{-15}$ m



$\simeq 10^4$ m

$n \simeq 10^{14}$ g.cm⁻³



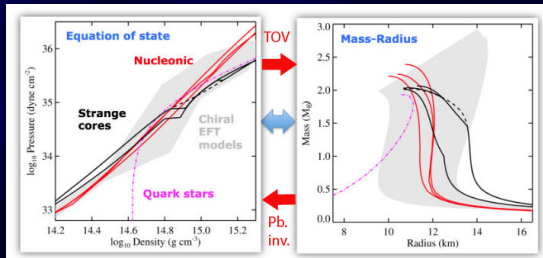
Link between EoS and M-R relation :

T.O.V equations (Tolmann-Oppenheimer-Volkov) :

$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r), \quad (1)$$

$$\frac{dP(r)}{dr} = -\frac{G\epsilon(r)m(r)}{r^2} \left(1 + \frac{P(r)}{\epsilon(r)c^2}\right) \left(1 + \frac{4\pi P(r)r^3}{m(r)c^2}\right) \times \left(1 - \frac{r_{sh} m(r)}{r M}\right)^{-1}. \quad (2)$$

Equation of state : $P(\epsilon) \leftrightarrow$ mass-radius relation.



[Anna Watts et al, 2014]

Masses distribution of NSs :

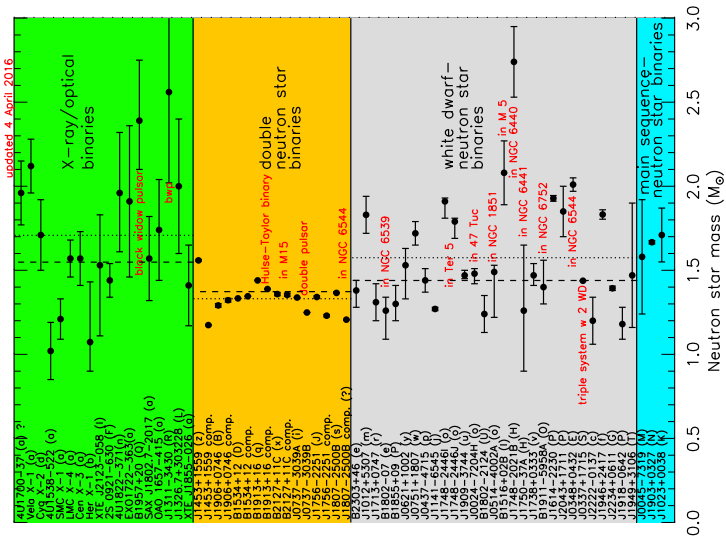


Figure : Jim Lattimer, stellarcollapse.org/nsmasses

Radii distribution of NSs :

Recent works :

$R \in [9; 15]$ km for a $1.4 M_{\odot}$ NS

Tools :

- **Thermal emission of NSs** (Guillot et al. 2013, Ozel et al. 2016, Bogdanov et al. 2016, Steiner et al. 2018) – (This talk)
- **X-ray bursts** (Poutanen et al. 2013, Ozel et al. 2016, Nattila 2017)
- **Gravitational waves from NS-NS mergers** (Tews et al. 2018)
- **NICER mission** (see Sebastien's talk)
- Future : **Athena mission** (Barcons et al. 2017)

The EoS for neutron stars interior :

Various kinds of EoS :

- Purely nucleonic \rightarrow no phase transition, **smooth EoS** with n, y_e, T .
- Phase transition :
 - hadronic matter : onset of hyperons, pion condensate ...
 - pure quark stars (absolutely stable strange matter) ?
 - hybrid stars (QGP/color superconductivity in the core) ?

Motivation for choosing **pure nucleonic matter** :

- **extrapolation** of **nuclear physics** knowledge (and uncertainties) towards high densities and low Y_p .
- Define M-R boundaries for smooth EoS to **Confront** with **observational data**.

The EoS model based on empirical parameters :

Definition of the empirical parameters :

$$\epsilon(n, \delta) = \epsilon_{IS} + \delta^2 \epsilon_{IV} \quad (3)$$

$$\epsilon_{IS} = E_{sat} + \frac{1}{2} K_{sat} x^2 + \frac{1}{3!} Q_{sat} x^3 + \frac{1}{4!} Z_{sat} x^4 + o(x^5) \quad (4)$$

$$\epsilon_{IV} = E_{sym} + L_{sym} x + \frac{1}{2} K_{sym} x^2 + \frac{1}{3!} Q_{sym} x^3 + \frac{1}{4!} Z_{sym} x^4 + o(x^5) \quad (5)$$

Taylor expansion around n_{sat} for the energy density :

$$\epsilon(n, \delta) = t(n, \delta) + \sum_{\alpha \geq 0} v_{\alpha}(\delta) \frac{x(n)^{\alpha}}{\alpha!} u_{\alpha}^N(x), \quad \delta = \frac{(n_n - n_p)}{n}, \quad x = \frac{(n - n_{sat})}{3n_{sat}}$$

$v_{\alpha}(\delta) = v_{\alpha,IS} + v_{\alpha,IV} \delta^2$, and $u_{\alpha}^N(x)$ ensure the convergence.

[Margueron et al. PRC 2018]

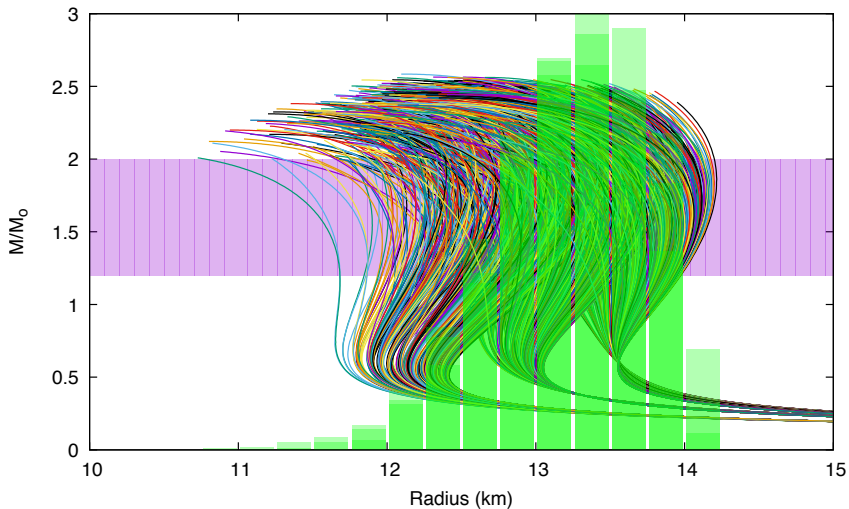
Our present knowledge of the empirical parameters :

Empirical parameters for various effective approaches :

		fixed			Explore in small interval		Explore in large interval				
Model (N_α)	der. order	E_{sat} MeV	E_{sym} MeV	n_{sat} fm^{-3}	L_{sym} MeV	K_{sat} MeV	K_{sym} MeV	Q_{sat} MeV	Q_{sym} MeV	Z_{sat} MeV	Z_{sym} MeV
		0	0	1	1	2	2	3	3	4	4
Skyrme (16)	Average	-15.88	30.25	0.1595	47.8	234.2	-129.8	-357.0	377.9	1500.0	-2218.9
	σ	0.15	1.70	0.0011	16.8	10.2	66.0	22.4	110.3	169.3	617.6
Skyrme (35)	Average	-15.87	30.82	0.1596	49.6	237.3	-131.7	-349.0	370.0	1447.6	-2175.1
	σ	0.18	1.54	0.0039	21.6	26.6	89.1	88.5	187.9	510.4	1069.4
RMF (11)	Average	-16.24	35.11	0.1494	90.2	268.0	-4.6	-1.9	271.1	5058.3	-3671.8
	σ	0.06	2.63	0.0025	29.6	33.5	87.7	392.5	357.1	2294.1	1582.3
RHF (4)	Average	-15.97	33.97	0.1540	90.0	248.1	128.2	389.2	523.3	5269.1	-9955.5
	σ	0.08	1.37	0.0035	11.1	11.6	51.1	350.4	236.8	838.4	4155.7
Total (50)	Average	-16.03	33.30	0.1543	76.6	251.1	-2.7	12.7	388.1	3925.0	-5267.5
	σ_{tot}	0.20	2.65	0.0054	29.2	28.6	131.7	431.1	289.4	2269.7	4281.9
	Min	-16.35	26.83	0.1450	9.9	201.0	-393.9	-748.0	-86.2	-903.0	-16916.2
	Max	-15.31	38.71	0.1746	122.7	355.4	213.2	949.8	846.3	9997.3	-4.9

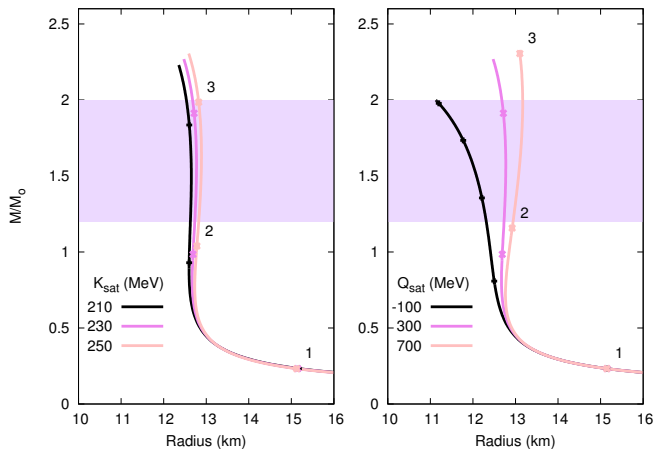
Dependence of M-R on the empirical parameters :

+ constrains : $0 < v_s^2 < c^2$ and $\epsilon_{IV}(n) > 0$ for $n < 4n_{sat}$



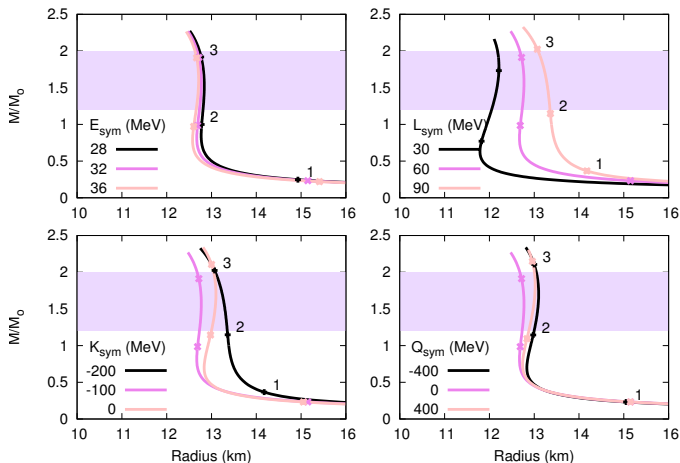
[Baillot d'Étivaux, Thesis, 2018]

Dependence of M-R on the empirical parameters : isoscalar channel



[Margueron et al. PRC 2018]

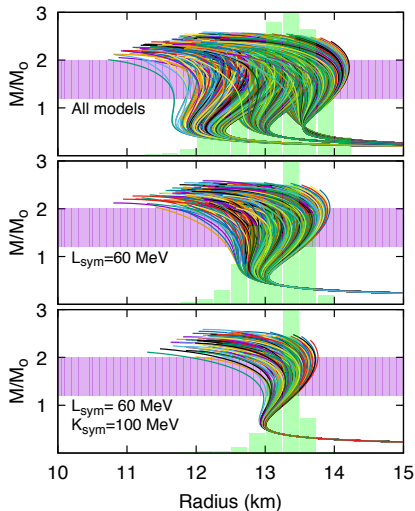
Dependence of M-R on the empirical parameters : isovector channel



[Margueron et al. PRC 2018]

Influence of the parameters on the M-R relation :

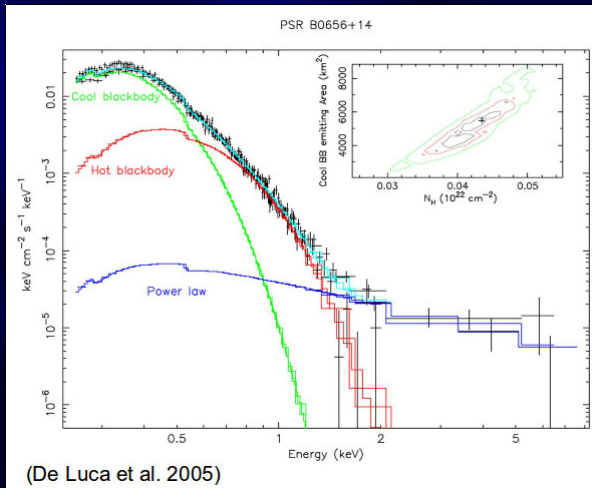
- Present knowledge of nuclear physics + smooth EoS
→ $R \simeq 11.5 - 14$ km and $M < 2.5M_{\odot}$
- Main source of uncertainties :
→ L_{sym} , K_{sym} and Q_{sat} .
- If better knowledge on L_{sym} and K_{sym} :
reduce uncertainty $\simeq 1$ km
- Measurement of R out of these boundaries :
→ non-smooth EoS
→ **Phase transition !**



Modeling the thermal emission of neutron stars using the meta-model

X ray thermal emission of neutron stars

Black body like emission : $F \propto T^4 (R_\infty/D)^2$ [Rutledge et al. 1999]



Application of the meta-model to NSs thermal emission

- **7 low mass X-ray transients :**
Constant flux, purely H atmosphere,
Low magnetic fields \rightarrow almost pure thermal components,
In globular clusters \rightarrow well constrained distances.
- **A single EoS \rightarrow simultaneous analysis.**
- **Theory directly implemented in the data analysis (first time !) :**
 - Observational parameters $\{P_\alpha\} = M, D, T, N_H, \dots$
 - Nuclear parameters $\{P_\beta\} = L_{sym}, K_{sym}, \dots$
 - EoS($\{P_\beta\}, M$) $\rightarrow R$
 - $\{P_\alpha\} + \{P_\beta\} \rightarrow$ **Spectral model**

≈ 50 free parameters for ≈ 1000 degrees of freedom.
 \rightarrow MCMC, Bayesian methods

Modeling the spectra with Xspec

Spectrum model used :

"pile-up" (Davis 2001, Bogdanov 2016), "TBgas" absorption and "nsatmos" for the atmosphere (Heinke et al. 2006) + "power-law".

Parameters which are allowed to vary :

- Pile-up α parameter
- Hydrogen column density on the line of sight $n_{H,22}$ (10^{22} cm^{-2})
- The power-law normalization.
- Distance to the stars D (kpc), surface effective temperature T_{eff} (K), the mass of the stars M (M_{\odot}).
- The nuclear empirical parameters $\{P_{\alpha}\} : \text{EoS}(\{P_{\alpha}\}, M) \rightarrow R$

Globular Clusters with qLMXBs and distances estimation

Source	distance (recent pub) (kpc)	distance (GAIA DRII-2018) (kpc)
47-Tuc	4.53 ± 0.08	4.50 ± 0.06
ω -Cen	4.59 ± 0.08	5.20 ± 0.09
M13	7.1 ± 0.62	7.9 ± 0.62
M28	5.5 ± 0.3	5.50 ± 0.13
M30	8.2 ± 0.62	8.10 ± 0.12
NGC6397	2.51 ± 0.07	2.30 ± 0.05
NGC6304	6.22 ± 0.26	5.90 ± 0.14

Prior distribution for the distances

Multiplying the likelihood by gaussian priors :

$$\frac{1}{N} \exp\left(-\frac{\chi^2}{2}\right) \longrightarrow \frac{1}{N} \exp\left(-\frac{\chi^2}{2}\right) \times \prod_i P^i(d^i),$$

with reference distances d_{ref}^i , and reference uncertainties $\sigma_{d,ref}^i$:

$$P^i(d^i) = \exp\left(-\frac{1}{2} \times \frac{d^i - d_{ref}^i}{\sigma_{d,ref}^i}\right)^2.$$

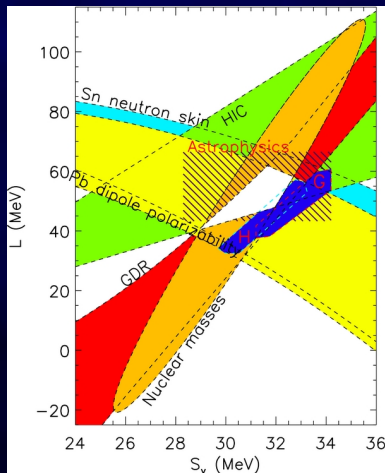
Constraining L_{sym} from nuclear physics knowledge

$$L_{sym} = 50 \pm 10 \text{ MeV} :$$

$$P(L_{sym}) = \exp\left(\frac{L_{sym} - 50}{2 \times 10}\right)^2$$

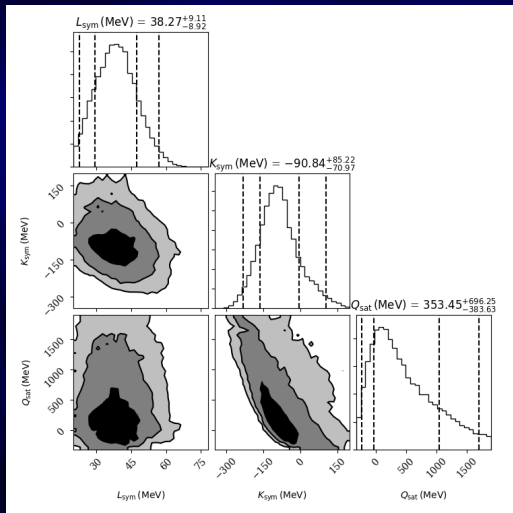
$$K_{sym} \in [-400 : 200] \text{ MeV}$$

$$Q_{sat} \in [-1300 : 1900] \text{ MeV}$$



[Lattimer and Lim 2013]

Constraints L_{sym} , K_{sym} and Q_{sat}

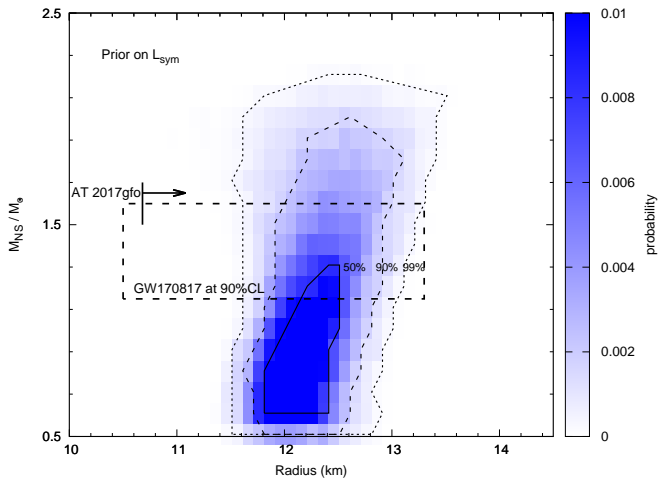


Tests of sensibility

Sources	Distances	prior L_{sym}	L_{sym} MeV	K_{sym} MeV	Q_{sat} MeV
all	Gaia DR2	yes	$37.2^{+9.2}_{-8.9}$	-85^{+82}_{-70}	318^{+673}_{-366}
all	qLMXB publ.	yes	$38.3^{+9.1}_{-8.9}$	-91^{+85}_{-71}	353^{+696}_{-484}
all	qLMXB publ.	yes	$38.6^{+9.2}_{-8.7}$	-95^{+80}_{-36}	-
all	qLMXB publ.	no	$27.9^{+11.4}_{-5.8}$	-60^{+107}_{-79}	498^{+738}_{-479}
all	qLMXB publ. 3σ	yes	$40.2^{+9.5}_{-9.2}$	-86^{+111}_{-85}	555^{+740}_{-494}
High S2N	Gaia DR2	yes	$37.5^{+9.0}_{-8.9}$	-88^{+76}_{-70}	263^{+764}_{-361}
Low S2N	Gaia DR2	yes	$50.3^{+9.8}_{-9.6}$	-1^{+134}_{-143}	881^{+671}_{-705}
all/47-Tuc	Gaia DR2	yes	$43.4^{+9.7}_{-9.3}$	-66^{+137}_{-102}	622^{+763}_{-560}
all/NGC6397	Gaia DR2	yes	$42.6^{+9.9}_{-9.5}$	-77^{+129}_{-96}	623^{+757}_{-544}
all/M28	Gaia DR2	yes	$42.5^{+9.5}_{-9.5}$	-80^{+124}_{-91}	597^{+717}_{-510}

[Baillot d'Étivaux et al. (to be submitted)]

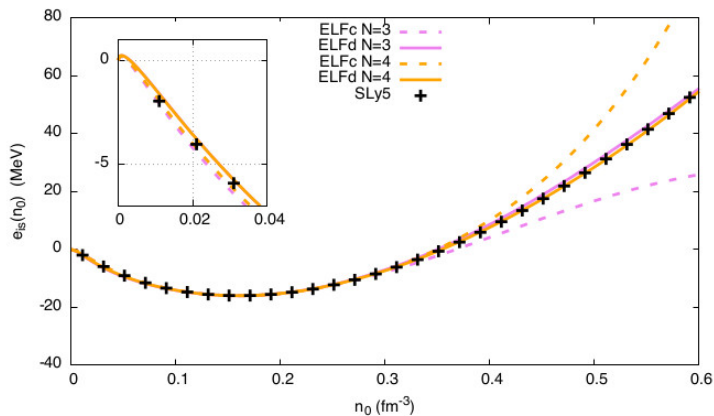
Constraining L_{sym} , K_{sym} and Q_{sat} : M-R distribution



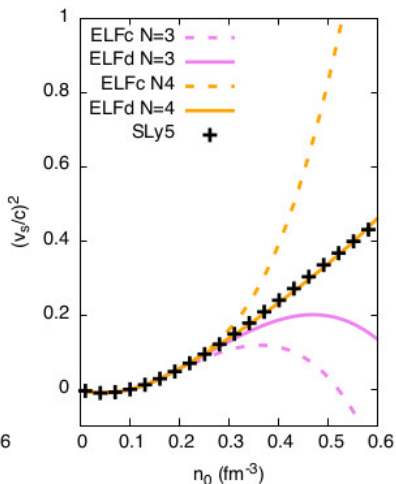
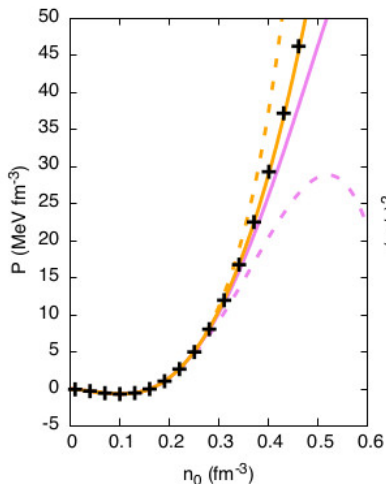
Conclusions

- Link between the EoS and the M-R relation.
- Nuclear EoSs can hardly produce radii below 11.5 km.
- Nucleonic models are able to reproduce observed data.
- The nuclear parameters which have the most impact on the EoS are : L_{sym} , K_{sym} , and Q_{sat} .
- NSs observation can bring new constraints on these parameters.

Ability of the EoS to mimic known EoS : example of SLy5



Ability of the EoS to mimic known EoS : example of SLy5



Parameters for isoscalar channel :

$$E_0 = e_b(x = 0, \delta = 0), \quad (6)$$

$$P_0 = n_{sat}^2 (1 + 3x)^2 \frac{\partial}{\partial x} e_b(x, \delta) \Big|_{x=0, \delta=0} = 0, \quad (7)$$

$$K_0 = \frac{\partial^2}{\partial x^2} e_b(x, \delta) \Big|_{x=0, \delta=0}, \quad (8)$$

$$Q_0 = \frac{\partial^3}{\partial x^3} e_b(x, \delta) \Big|_{x=0, \delta=0}, \quad (9)$$

$$Z_0 = \frac{\partial^4}{\partial x^4} e_b(x, \delta) \Big|_{x=0, \delta=0}. \quad (10)$$

Parameters for isovector channel :

$$S_{sym} = e_{s,iv}(x=0), \quad (11)$$

$$L_{sym} = \left. \frac{\partial}{\partial x} e_{s,iv}(x) \right|_{x=0, \delta=0}, \quad (12)$$

$$K_{sym} = \left. \frac{\partial^2}{\partial x^2} e_{s,iv}(x) \right|_{x=0, \delta=0}, \quad (13)$$

$$Q_{sym} = \left. \frac{\partial^3}{\partial x^3} e_{s,iv}(x) \right|_{x=0, \delta=0}, \quad (14)$$

$$Z_{sym} = \left. \frac{\partial^4}{\partial x^4} e_{s,iv}(x) \right|_{x=0, \delta=0}. \quad (15)$$

Spectrum 1 :

