Constraints on realistic EoS
IST EoS
Modelling of neutron stars
Conclusions

Unified equation of state consistent with astrophysical, gravitational, high- and low- energy nuclear physics data

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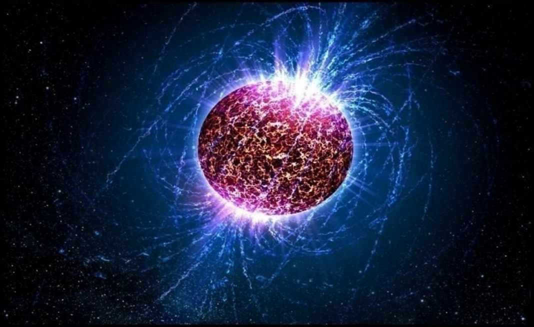

MODE-SNR-PWN 2019, 8-10 Apr

Outline

1 Constraints on realistic EoS

2 IST EoS

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Strongly Interacting Matter Phase Diagram

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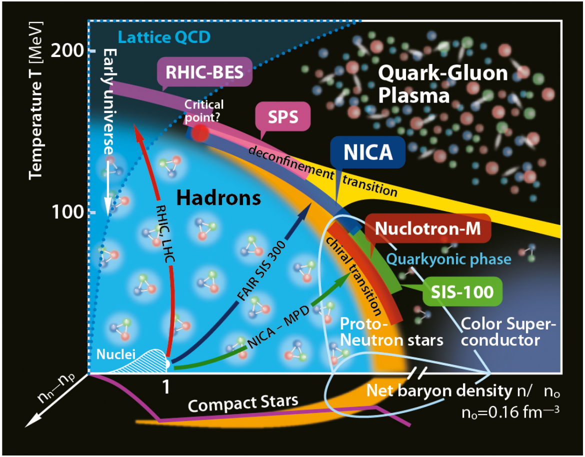
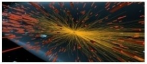


Constraints on the EoS

HEP



- proton flow
- hadron multiplicities

Astro

- 2.01(4) M_{sun}
- NSs cooling
- observations of pulsars

⇩

M-R relation

Nucl. Phys.

- nuclear matter ground state

Grav. Phys.

- NS+NS
- NS+BH

collisions

First NS+NS merger GW170817

General Requirements

- causality
- thermodynamic consistency
- multicomponent character (n, p, e, ...)
- electric neutrality
- β-equilibrium
- realistic interaction between the constituents

Constrains on EoS: nuclear matter ground state

- Lowest energy state $\Rightarrow$ no excitations
  zero temperature T = 0

- Dynamical equilibrium of nuclei and vacuum
  zero pressure p = 0

- Size of large nuclei $R_A \simeq r_0\, A^{1/3}$, $r_0 \simeq 1.15$ fm

  normal nuclear density $n_0 \simeq 0.16$ fm^{-3}

- Binding energy per nucleon $\simeq$ 16 MeV:

$$\frac{E_B}{A} = \frac{Am - E_A}{A} = m - \frac{\epsilon_0}{n_0}$$

  energy density at ground state $\epsilon_0 \simeq 148$ MeV fm^{-3}

$$K_0 = 9\,\frac{dp}{dn}$$

- Incompressibility factor = 200 - 260 MeV:

  E. Khan, Phys.Rev. C, 80, 011307 (2009); M. Dutra et al., Phys.Rev. C, 85, 035201 (2012)

- Symmetry energy at saturation density $E_{sym}(n = n_B) \equiv J = 30 \pm 4 MeV$
  Slope $L = 3n\frac{\partial E_{sym}}{\partial n}$ = 20 - 115 MeV
  Zhang, Z., Chen, L.-W., Phys. Lett. B, 726, 234 (2013)

Realistic EoS has to reproduce properties of ground state

Constrains on EoS: Astro/Gravity observations

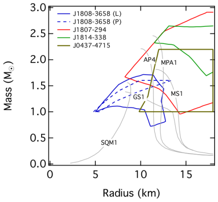
Maximal observed mass of NS

J1614-2230 pulsar: $\mathbf{M_{max} = 1.97(4)M_\odot}$
P. B. Demorest et al., Nature, 467, 1081 (2010)

PSR J0348-0432 pulsar: $\mathbf{M_{max} = 2.01(4)M_\odot}$
J. Antoniadis et al., Science, 340, 448 (2013)

Observation of NS - ... binary systems

Shapiro Delay and radio timing constrain
R - M dependence of NSs
F. Özel, P. Freire, A.& A., 54, 401 (2006)



Coalescence of NS - ... binary systems

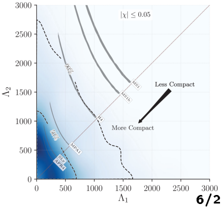
LIGO and Virgo collaborations, ApJL, 848, L12 (2017)

LIGO and Virgo collaborations, PRL 119, 161101 (2017)

First NS+NS merger GW170817

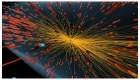
Love numbers and tidal polarizability are
highly sensitive to EoS

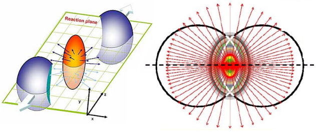
Ch. C. Moustakidis et al., Phys. Rev. C 95, 059904 (2017)



Realistic EoS has to fulfil these constraints

Constrains on EoS: Flow constraint





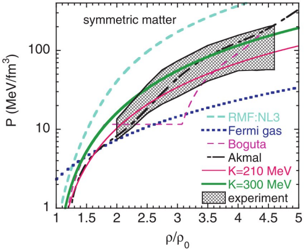
Non-central necleus-nucleus collisions

asymmetry of spatial distribution of nucleons at initial moment leads to strong anisotropy of particle flows

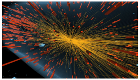
Hydrodynamic reconstruction of flows

anisotropic expansion is caused by gradient of pressure, which gives an access to EoS

P. Danielewicz, R. Lacey, W. G. Lynch, Science 298, 1593 (2002)



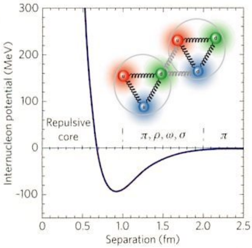
Constrains on EoS: hard core radius of nucleons



Nucleon-nucleon scattering

hard core radius of nucleons is extracted as a parameter of microscopic interaction potential: **R ≃ 0.5 fm**

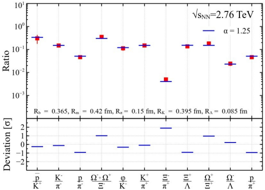
A. Bohr and B. Mottelson, Nucl. Structure Benjamin, NY, 266 (1969)
M. Naghdi, Phys. Part. Nucl. 5, 924 (2014)



Hadron production in heavy ion collisions

hard core radii of hadrons control the rate of their production in thermal medium: **R ≃ 0.3 - 0.5 fm**

A. Andronic et al., Nucl. Phys. A 772, 167 (2006)
K. A. Bugaev et al., Nucl. Phys. A 970, 133 (2018)
V. V. Sagun et al., EPJ Web Conf., 137, 09007 (2017)



EoS with hard core repulsion

- Hard core reduces volume available for motion of particles by $V_{excl} = Nb$

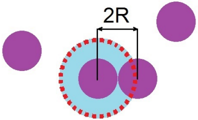
$$V \to V - V_{excl} \;\Rightarrow\; \overbrace{p = nT}^{ideal\ \ gas\ \ EoS} = \frac{NT}{V} \to \frac{NT}{V - V_{excl}} = \frac{NT}{V - Nb} = \overbrace{\frac{nT}{1 - nb}}^{Van\ \ der\ \ Waals\ \ EoS}$$

- Van der Waals EoS ($b = const$) in the Grand Canonical Ensemble

$$\begin{cases} p = p(T, \mu) \\ n = \frac{\partial p}{\partial \mu} \end{cases} \;\Rightarrow\; p = T \int_{\vec{k}} \exp\left(\frac{\mu - pb - \sqrt{m^2 + k^2}}{T} \right) = p_{id}(T, \mu - pb)$$

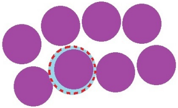
- Excluded volume (per particle) depends on density ($b \neq const$)

Low densities

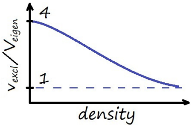


$b \simeq \frac{1}{2} \cdot \frac{4\pi}{3} (2R)^3 = 4v$

High densities



$b \simeq v$



The Induced Surface Tension (IST) EoS

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- Quantitites of the Boltzmann ideal gas

$$p = nT, \quad n = \sum_i n_i^{id}, \quad n_i^{id} = \frac{p_i^{id}}{T}$$

- Virial expansion for one particle species

$$\frac{p}{T} = n + a_2 n^2 + a_3 n^3 + a_4 n^3 + \dots$$

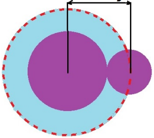
Ideal gas: $a_2 = 0,\ a_3 = 0,\ a_4 = 0,\ \dots$

Hard spheres: $a_2 = 4v,\ a_3 = 10v^2,\ a_4 = 18.365v^3,\ \dots$

R.K.Pathria, Statistical Mechanics, Pergamon Press, Oxford, 1972

- Excluded volume at low density ($v_i = \frac{4\pi}{3} R_i^3$ and $s_i = 4\pi R_i^2$)

$$a_2^{ij} = \frac{1}{2} \cdot \frac{4\pi}{3} (R_i + R_j)^3 = \frac{1}{2} \cdot (v_i + s_i R_j + R_i s_j + v_j)$$



- Virial expansion for many particle species

$$\frac{p}{T} = \overbrace{\sum_i n_i^{id}}^{\simeq n} - \overbrace{\sum_{i,j} a_2^{ij} n_i^{id} n_j^{id}}^{\simeq a_2 n^2} + \dots = \sum_i \frac{p_i^{id}}{T} \Big(1 - \overbrace{v_i \sum_j \frac{p_j^{id}}{T}}^{bulk\ term} - \overbrace{s_i \sum_j \frac{p_j^{id}}{T} R_j}^{surface\ term} \Big) + \dots$$

EoS with induced surface tension (IST)

$$p=\sum_i p_i^{id}\Big(1-\overbrace{\frac{v_i}{T}\underbrace{\sum_j p_j^{id}}_{p+\mathcal{O}(n^2)}}^{bulk\ \ term}-\overbrace{\frac{s_i}{T}\underbrace{\sum_j p_j^{id}R_j}_{\Sigma+\mathcal{O}(n^2)}}^{surface\ \ term}\Big)+\ldots$$

- Σ is conjugated to s_i – **induced surface tension (IST)**

$$\begin{cases} p=\sum\limits_i p_i^{id}\left(1-\frac{pv_i}{T}-\frac{\Sigma s_i}{T}\right)\\ \Sigma=\sum\limits_i p_i^{id}\left(1-\frac{pv_i}{T}-\frac{\alpha\Sigma s_i}{T}\right)R_i\end{cases}$$

α accounts for not uniqueness of extrapolation to high densities

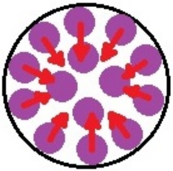
- High density extrapolation (gives exponentials)

$$\begin{cases} p=\sum\limits_i p_i^{id}\exp\left(-\frac{pv_i+\Sigma s_i}{T}\right)\\ \Sigma=\sum\limits_i p_i^{id}\exp\left(-\frac{pv_i+\alpha\Sigma s_i}{T}\right)R_i\end{cases}\rightarrow\begin{cases} p=\sum\limits_i p_i^{id}(\mu_i-pv_i-\Sigma s_i)\\ \Sigma=\sum\limits_i p_i^{id}(\mu_i-pv_i-\alpha\Sigma s_i)R_i\end{cases}$$

VS, A. Ivanytskyi, K. Bugaev, I. Mishustin, NPA 924, 24 (2014)

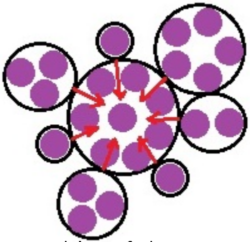
Physical Origin of the Induced Surface Tension

Vacuum



attraction of constituents
⇒ **eigen surface tension**

Medium



repulsion of clusters
⇒ **induced surface tension**

- Hard core repulsion only in part is accounted by eigen volume
- The rest corresponds to surface tension and curvature tension
  Curvature tension can be accounted explicitly or implicitly
- Physical clusters tend to have spherical (in average) shape

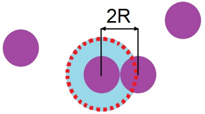
Determination of α

- One component EoS with IST and $\alpha > 1$

$$\begin{cases} p = p^{id} \exp\left(-\frac{pv+\Sigma s}{T}\right) \\ \Sigma = p^{id} \exp\left(-\frac{pv+\alpha\Sigma s}{T}\right) R \end{cases} \Rightarrow \begin{cases} p = p^{id} \exp\left(-\frac{pb}{T}\right) R \\ \Sigma = pR \exp\left(\frac{(1-\alpha)\Sigma s}{T}\right) \end{cases}$$

VVS, A. I. Ivanytskyi, K. A. Bugaev, I. N. Mishustin, Nucl. Phys. A, 924, 24 (2014)

Excluded volume: $\frac{b}{v} = 1 + 3e^{\frac{(1-\alpha)\Sigma s}{T}} \to \begin{cases} 4, & \Sigma \to 0 \\ 1, & \Sigma \to \infty \end{cases}$



$\alpha > 1$ **switches different regimes of excluded volume**

- Virial expansion of one component EoS with IST

Second virial coefficient: $a_2 = 4V$ is reproduced always

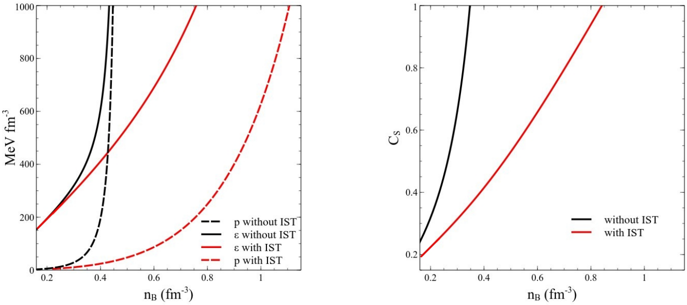
Third virial coefficient: $a_3 = 10V^2 \Rightarrow \alpha = \frac{4}{3}$
a_4 is not reproduce

Fourth virial coefficient: $a_4 \simeq 18.365 V^3 \Rightarrow \alpha \simeq 1.245$
a_3 – reproduced with 16% accuracy

$\alpha > 1.245$ **reproduces two (3rd and 4th) virial coefficients**

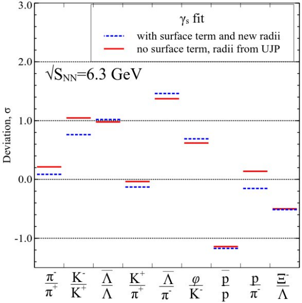
Effect of the IST

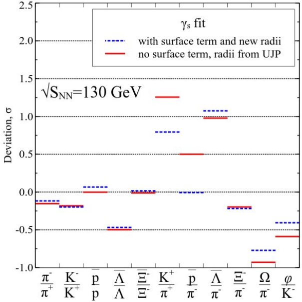
Thermodynamic parameters with and without IST



Hadron Resonance Gas Model

- Hadrons with masses $\leq$ 2.5 GeV (widths, strong decays, zero strangeness)
- 111 independent particle ratios measured at 14 energies (from 2.7 GeV to 200 GeV)
- 14 $\times$ 4 local parameters ($T, \mu_B, \mu_{I3}, \gamma_s$) + 5 global parameters (hard core radii)





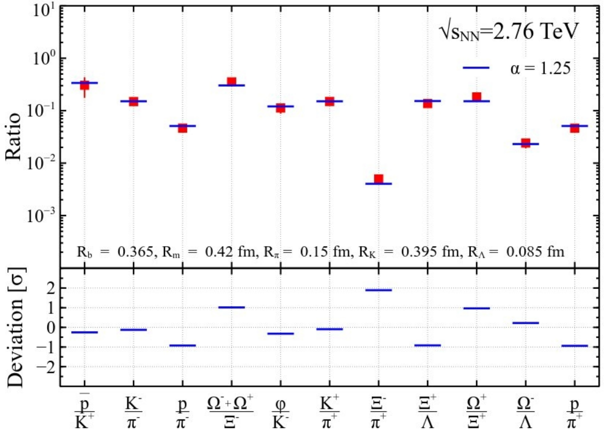
$R_b = 0.365\ fm,\ R_m = 0.42\ fm,\ R_\pi = 0.15\ fm,\ R_K = 0.395\ fm,\ R_\Lambda = 0.085\ fm$

$$\text{Overall}\ \chi^2/dof \simeq 1.038$$

K.A. Bugaev, et al., NPA 970, p. 133-155, (2018)
VVS, Ukr. J. Phys. 59, 755 (2014)

Hadron Resonance Gas at ALICE Energies

- 11 independent particle yields, 6 parameters (temperature + 5 hard core radii)
- Overal $\chi^2/dof \simeq 0.89$
- Freeze out temperature $T_{FO} = 148 \pm 7$ MeV



The Induced Surface Tension (IST) EoS

$$\begin{cases} p = \sum_{i}^{all\ particles} \left[p_{id}(T, \mu_i - pV_i - \Sigma S_i + U_{at} \pm U_{sym}) + p_{id}(\mu_e) - p_{at} + p_{sym} \right] \\ \Sigma = \sum_{i}^{all\ particles} p_{id}(T, \mu_i - pV_i - \alpha \Sigma S_i + U_0) R_i \end{cases}$$

p_{id} – pressure of the ideal gas for quantum statistics
Σ – induced surface tension
U_0, α – model parameters

$$Thermodynamic\ consistency\ of\ the\ model: \quad \frac{\partial p_{int}}{\partial n_{id}} = n_{id} \frac{\partial U(n_{id})}{\partial n_{id}}$$

$$Parametrization\ of\ the\ mean\ field\ potential: U_{at} = -C_d^2 n_{id}^{\kappa}$$

VVS, et al., Nucl. Phys. A, 924, 24 (2014)
A. Ivanytskyi et al., PRC 97, 064905 (2018)

Mean field interaction for nuclear matter

- Thermodynamic consistency provides identity $\frac{\partial p}{\partial \mu} = n$

$$p(\mu) = p^{id}(\mu - U(x)) + p_{int}(x), \quad p_{int}(x) = \int_0^x dx' x' \frac{\partial U(x')}{\partial x'}$$

x – any quantity (density, asymmetry parameter, ...)

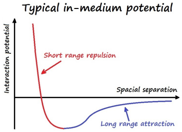
K. A. Bugaev and M. I. Gorenstein, Z. Phys. C 43, 261 (1989)

D.H. Rischke, et al. Z. Phys. C 51, 485 (1991)

Long range attraction (negative contribution to pressure)

$$p_{at}(x) = -\frac{\kappa}{\kappa+1} C_d^2 x^{1+\kappa}, \quad x = n_n^{id} + n_p^{id}$$

$\kappa < 1, \ C_d^2$ – fitted to flow constraint and properties of ground state
A. Ivanytskyi et al., PRC 97, 064905 (2018)



- Repulsion due to symmetry energy (positive contribution to pressure)

binding energy of stable nuclei: $E_{sym} = a_{sym} \frac{(N-Z)^2}{A}, \ a_{sym} = 30 \pm 4 \ MeV$

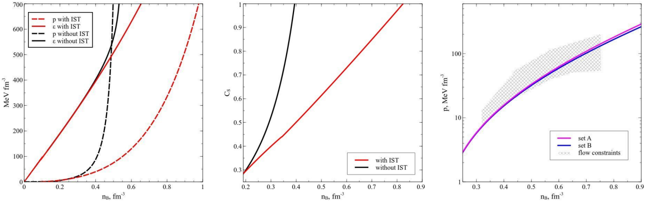
$$p_{sym}(x) = \frac{A_{sym} x^2}{1 + (B_{sym} x)^2}, \quad x = n_n^{id} - n_p^{id}$$

$A_{sym}, \ B_{sym}$ – fitted to a_{sym} and slope of symmetry energy at ground state

EoS for NS interiors

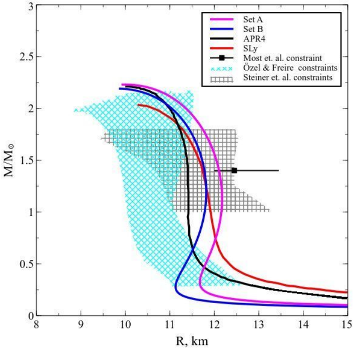
$$\begin{cases} p = \sum\limits_{i}^{all\ \ particles} \left[p_{id}(T, \mu_i - pV_i - \Sigma S_i + U_{at} \pm U_{sym}) + p_{id}(\mu_e) - p_{at} + p_{sym} \right] \\ \Sigma = \sum\limits_{i}^{all\ \ particles} p_{id}(T, \mu_i - pV_i - \alpha \Sigma S_i + U_0) R_i \end{cases}$$

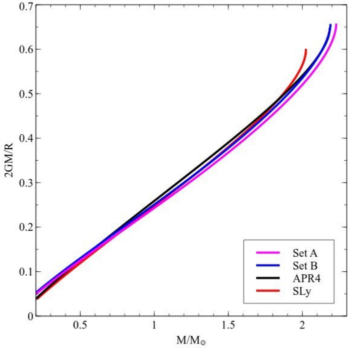
- **Degrees of freedom:** neutrons, protons, electrons
- **Quantum statistics** is accounted by construction of p_{id} and n_{id}
- **Realistic interaction**: HC repulsion, MF attraction, symmetry energy repulsion
- **Virial coefficients** of classic hard spheres are reproduced
- **Causal behaviour** up to densities where QGP is expected and **Flow constraint**



VVS, I. Lopes, A. Ivanytskyi, APJ, 871, 157 (2019)
VVS, I. Lopes, ApJ, 850, 75 (2017)

M-R relation and compactness of NS





IST EoS PARAMETERS

IST EoS Sets	r fm	α —	κ —	B^{sym} fm^3	A^{sym} $MeV \cdot fm^3$	C_d^2 $MeV \cdot fm^{3\kappa}$	U_0 MeV	K_0 MeV	J MeV	L MeV	M_{max} $M_\odot$
A (magenta curve on Fig)	0.492	1.245	0.263	3.5	16.896	143.564	147.456	200.03	30.0	114.91	2.229
B (blue curve on Fig)	0.484	1.245	0.26	4.5	14.762	144.042	150.97	200.00	30.0	113.28	2.189

The NS core was modelled within the IST EoS, while its crust was described via the polytropic EoS with $\gamma = \frac{4}{3}$

VVS, I. Lopes, A. Ivanytskyi, APJ, 871, 157 (2019)

Summary

- Using a novel a thermodynamically self-consistent IST EoS the properties of the NS at zero-temperature limit were calculated.
- It was shown that the present EoS can be successfully applied to the description of the hadron multiplicities measured in A+A collisions, to studies of the nuclear matter phase diagram and to modelling of the NS interiors.
- IST EoS satisfies all astrophysical constraints, correctly reproduce properties of normal nuclear density, proton flow data, hadron multiplicities measured in A+A collisions and nuclear matter properties near the (3)CEP.
- The description of the compact stars with the IST provides with a strong constraint on the attraction contribution in EoS at zero temperature.
- The IST EoS gives a possibility to describe the strongly interacting matter phase diagram in a wide range of its thermodynamic parameters which helps to create a solid bridge between the astrophysical data, high-energy nuclear physics and gravitation physics.

Strongly Interacting Matter Phase Diagram

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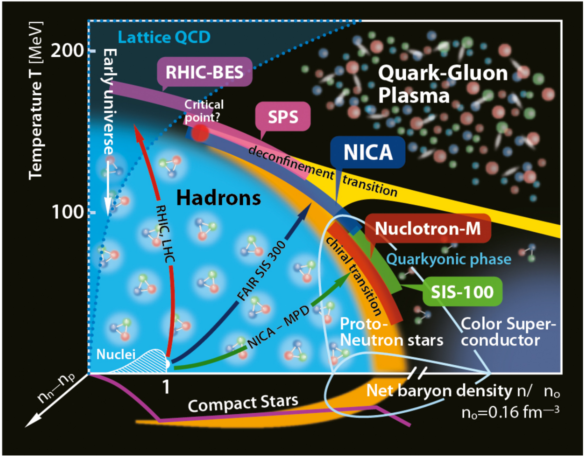

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